Tests of proportions

ST551 Lecture 11

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One-sided confidence intervals

Confidence intervals

We motivated CI's as all values of μ_0 that would not be rejected in a **two-sided** hypothesis test of $H_0 : \mu = \mu_0$.

Two-sided p-values, **two-sided** rejection regions and **two-sided** confidence intervals are generally equivalent:

$$
\begin{aligned}
p < \alpha &\iff \text{Reject } H_0 : \mu = \mu_0 \text{ at level } \alpha \\
&\iff \mu_0 \text{ is outside} (1-\alpha) 100\% \text{ confidence interval} \\
p > \alpha &\iff \text{Fail to reject } H_0 : \mu = \mu_0 \text{ at level } \alpha \\
&\iff \mu_0 \text{ is inside} (1-\alpha) 100\% \text{ confidence interval}
\end{aligned}
$$

One-sided confidence intervals

You can motivate them from one-sided tests too.

You end up with an infinite bound on one end.

So, why not report one sided CIs?

You don't always do a hypothesis test. A plausible range for a parameter value should be two-sided. (If there isn't a value of interest, how could there be a direction of interest?)

Should a plausible range for depend on *your* hypothesis of interest? More useful for others to give a 95% two-sided interval.

Yes, this means your one-sided test might not agree with your two-sided confidence interval.

Should we ever do one-sided tests? Some people argue "No, we should never do one sided tests". I'd say, you can, but you better have a really good reason, or someone will accuse you of doing it just to get a smaller p-value.

Binomial Proportions

Data Setting

Population: $Y \sim \text{Bernoulli}(p)$, i.e.

$$Y = \begin{cases} 1, & \text{with probability } p \\ 0, & \text{with probability } 1 - p \end{cases}$$

$E(Y) = p$, $Var(Y) = p(1-p)$ When mean and variance share parameters we say there is a **mean-variance relationship**.

Parameter: $\mu = E(Y) = p$, the population proportion

Sample: n i.i.d from population: $Y_1, \ldots, Y_n$

Statistic: $\overline{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i = \hat{p}$, the sample proportion.

Testing

Null hypothesis: $H_0 : p = p_0$

1. Exact test: use fact that

$$n\overline{Y} \sim \text{Binomial}(n, p)$$

2. Approximate test: use fact that

$$\overline{Y} \dot{\sim} N\left(E(Y), \frac{Var(Y)}{n}\right) = N\left(p, \frac{p(1-p)}{n}\right)$$

Exact Binomial Test:

Complete worksheet (Charlotte will provide)

1. Get into groups according to number on worksheet at numbered whiteboard
2. Write answers to bold questions on whiteboard as you complete them (so I can see where you are up to)