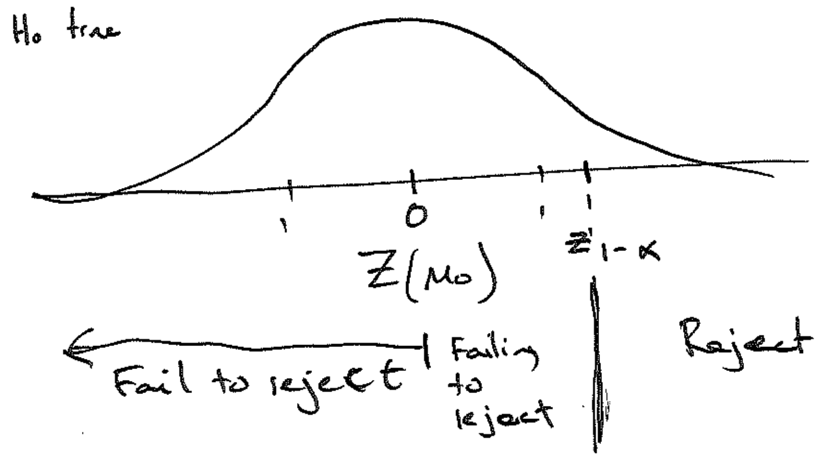
One-sided confidence intervals

You can motivate them from one-sided tests too.

Our CI will be values μ_0 where we would fail to reject $H_0: \mu = \mu_0$

One-sided $H_A: \mu > \mu_0 \implies$ Reject H_0 when $Z(\mu_0) > z_{1-\alpha}$



CI

all μ_0 s.t.

$$Z(\mu_0) < z_{1-\alpha}$$

$$\frac{\bar{Y} - \mu_0}{\sigma/\sqrt{n}} < z_{1-\alpha}$$

$$-\mu_0 < z_{1-\alpha}\,\sigma/\sqrt{n} - \bar{Y}$$

$$\mu_0 > \bar{Y} - z_{1-\alpha}\,\sigma/\sqrt{n}$$

You end up with an infinite bound on one end.

$(1-\alpha)100\%$ CI

$$\left(\bar{Y} - z_{1-\alpha}\frac{\sigma}{\sqrt{n}}\ ,\ \infty\right)$$

Confidence intervals

We motivated CI's as all values of μ_0 that would not be rejected in a **two-sided** hypothesis test of $H_0 : \mu = \mu_0$.

Two-sided p-values, **two-sided** rejection regions and **two-sided** confidence intervals are generally equivalent:

$$
\begin{aligned}
p < \alpha \iff & \text{Reject } H_0 : \mu = \mu_0 \text{ at level } \alpha \\
& \qquad\qquad (1-\alpha)100\% \\
\iff & \mu_0 \text{ is outside } \cancel{95}\% \text{ confidence interval} \\
p > \alpha \iff & \text{Fail to reject } H_0 : \mu = \mu_0 \text{ at level } \alpha \\
\iff & \mu_0 \text{ is inside } \cancel{95}\% \text{ confidence interval} \\
& \qquad\qquad (1-\alpha)100\%
\end{aligned}
$$

EXACT BINOMIAL TEST

One-sided Upper

$H_0: p = p_0$ vs. $H_A: p > p_0$

- Rejection region

  Reject H_0 for $X = \sum_{i=1}^{n} Y_i > c$

  where c s.t. $P_{H_0}(X > c) \leq \alpha$

- p-value for observed $X = x$

$$p = \sum_{k=x}^{n} P_{H_0}(X = k)$$

One-sided Lower

$H_0: p = p_0$ vs. $H_A: p < p_0$

- Rejection region

  Reject H_0 for $X < c$

  where c s.t. $P_{H_0}(X < c) \leq \alpha$

- p-value for observed $X = x$

$$p = \sum_{k=0}^{x} P_{H_0}(X = k)$$

EXACT BINOMIAL TEST

Two-sided

$$H_0: p = p_0 \quad \text{vs.} \quad H_A: p \neq p_0$$

- <u>Rejection region</u>

  Reject H_0 for $P_{H_0}(X) \leq c$ where

  $$c \text{ s.t. } \sum_{k:\, P_{H_0}(X=k) \leq c} P_{H_0}(X=k) \leq \alpha$$

- <u>p-value</u> for observed $X = x$

  $$p = \sum_{k:\, P_{H_0}(X=k) \leq P_{H_0}(X=x)} P_{H_0}(X=k)$$

  i.e. sum of probabilities for all values, k, that are as or less likely to occur than x, under H_0.