# Sign Test

ST551 Lecture 13

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# Sign test

# Data Setting

**Population:** $Y \sim$ something with c.d.f $F_Y(y) = P(Y \leq y)$

**Parameter**: $M = F_Y^{-1}(0.5)$, the population median

**Sample:** $n$ i.i.d from population: $Y_1, \ldots, Y_n$

**Null hypothesis:** $H_0 : M = M_0$

# Your Turn:

Consider the hypothesis $H_0 : M = M_0$

Imagine transforming the $Y_i$, $i = 1, \ldots, n$ to

$$X_i = \begin{cases} 1, & Y_i \leq M_0 \\ 0, & Y_i > M_0 \end{cases}$$

**If the null hypothesis is true $M = M_0$, what is $P(X_i = 1)$?**

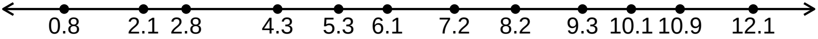
(You can assume $Y$ is a continuous distribution)

# Sign test

To test $H_0 : M = M_0$, perform a Binomial test on $X_i = \mathbf{1}\{Y_i \leq M_0\}$ with $H_0 : p = 0.5$.

# Example

Consider a sample, $n = 12$, with the sample values:




Consider testing $H_0 : M = 4$ versus a two-sided alternative $H_A : M \neq 4$ (at the $\alpha = 0.05$ level).

$X_i = \mathbf{1}\{Y_i \leq 4\}$

$\hat{p}_{M_0} = \frac{1}{n} \sum_{i=1}^{n} X_i = 0.25$

$Z(p_0 = 0.5) = \frac{\hat{p}_{M_0} - p_0}{\sqrt{p_0(1-p_0)/n}} = -1.73$

Compare to $z_{1-\alpha/2} = 1.96$

We **fail to reject** the null hypothesis.

# Your turn: 95% Confidence Interval

We can *invert the test* by considering all $M_0$ for which we would fail to reject the null hypothesis $H_0 : M = M_0$.

**Would you reject for the value on your slip of paper?**

**Why do we only need to consider the actual sample values?**

# Your turn: 95% Confidence Interval

```
##     m_0
## 1   0.8
## 2   2.1
## 3   2.8
## 4   4.3
## 5   5.3
## 6   6.1
## 7   7.2
## 8   8.2
## 9   9.3
## 10 10.1
## 11 10.9
## 12 12.1
```

95% confidence interval for $M$ is (      ,       )

# Confidence interval in general

Solve for $M_0$ that satisfy (i.e. not in rejection region)

$$\left|\frac{X/n - 0.5}{0.5\sqrt{n}}\right| < z_{1-\alpha/2}, \quad \text{where } X = \text{number of observations } \leq M_0$$

$$-z_{1-\alpha/2} < \frac{X/n - 0.5}{0.5/\sqrt{n}} < z_{1-\alpha/2}$$

$$n(0.5 - z_{1-\alpha/2}\frac{0.5}{\sqrt{n}}) < X < n(0.5 + z_{1-\alpha/2}\frac{0.5}{\sqrt{n}})$$
$$\frac{1}{2}(n - z_{1-\alpha/2}\sqrt{n}) < X < \frac{1}{2}(n + z_{1-\alpha/2}\sqrt{n})$$

# Confidence interval in general

So, $M_0$ is in interval if the number of observations smaller than $M_0$ is between:

$$\frac{1}{2}(n - z_{1-\alpha/2}\sqrt{n}) \quad \text{and} \quad \frac{1}{2}(n + z_{1-\alpha/2}\sqrt{n})$$

The smallest value that satisifies this is the

$$\left(\frac{1}{2}(n - z_{1-\alpha/2}\sqrt{n})\right)^{\text{th}} \text{ smallest observation}$$

The largest value that satisifies this is the

$$\left(\frac{1}{2}(n + z_{1-\alpha/2}\sqrt{n}) + 1\right)^{\text{th}} \text{ smallest observation}$$

# Confidence interval for median

Approximate (based on approximate Binomial test) confidence interval for the median:

$$\left(\left(\frac{n - z_{1-\alpha/2}\sqrt{n}}{2}\right)^{\text{th}} \text{ smallest observation}, \left(\frac{n + z_{1-\alpha/2}\sqrt{n}}{2} + 1\right)^{\text{th}} \text{ smallest observation}\right)$$

May need to round $(.)^{\text{th}}$ to nearest integers

# Example, continued

$n = 12, \alpha = 0.05 \implies$

$$\left(\left(\frac{12 - 1.96\sqrt{12}}{2}\right)^{\text{th}} \text{ smallest observation}, \left(\frac{12 + 1.96\sqrt{12}}{2} + 1\right)^{\text{th}} \text{ smallest observation}\right)$$

$$\left((2.61)^{\text{th}} \text{ smallest observation}, (10.40)^{\text{th}} \text{ smallest observation}\right)$$

$$\left(3^{\text{rd}} \text{ smallest observation}, 10^{\text{th}} \text{ smallest observation}\right)$$

$$(2.8, 10.1)$$

# Sign test for discrete distributions/data

1. Remove all values exactly equal to $M_0$
2. Proceed with test as usual (with a reduced sample size $n$)

# Sign test: exactness

**Finite sample exact?** No

- Discrete nature of data means we can't achieve a lot of signficance levels
- Normal approximation is only an approximation...

**Assymptotically exact?** Yes

# Sign test: consistency

The sign test test is consistent. Comes from Binomial test being consistent (which comes from Z-test being consistent).

## Next time...

Signed Rank test