ST551 Lecture 19 Nov 3 '17

What is $E\left(\frac{S_p^2}{n} + \frac{S_p^2}{m}\right)$? I.e. what is our estimated variance of $\bar{Y}-\bar{X}$ on average when $\sigma_y^2 \neq \sigma_x^2$

$$E\left(\frac{S_p^2}{n} + \frac{S_p^2}{m}\right) = E\left(\frac{m+n}{mn} S_p^2\right), \text{ rearrange to pull out } S_p^2$$

$$= \frac{m+n}{mn}\left[\frac{(n-1)\,E(S_y^2) + (m-1)\,E(S_x^2)}{n+m-2}\right], \text{ substitute in } S_p^2 \text{ \& linearity of expectation}$$

$$= \frac{m+n}{mn}\left[\frac{(n-1)\,\sigma_y^2 + (m-1)\,\sigma_x^2}{n+m-2}\right], \quad E(S_y^2)=\sigma_y^2,\ E(S_x^2)=\sigma_x^2$$

$$= \frac{\cancel{(m+n)}\cancel{(n-1)}}{(m\cancel{n})\cancel{(n+m-2)}}\sigma_y^2 + \frac{\cancel{(m+n)}\cancel{(m-1)}}{\cancel{(mn)}\cancel{(n+m-2)}}\sigma_x^2, \text{ expand}$$

$$\underset{\substack{\text{large } n \\ \text{large } m}}{\approx} \frac{\sigma_y^2}{m} + \frac{\sigma_x^2}{n}, \quad \text{for large } n,\ \frac{n-1}{n} \approx 1$$