Other two sample comparisons

ST551 Lecture 25

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Charlotte to ask about Weds lab

Labs on Weds : YES!

HW #7 due Friday @ midnight

HW #8 last homework released Weds night
due Fri @ midnight 1st Dec

Some time next week : practice final
final study guide

Charlotte will have office hours in finals week.

Final : Thu Dec 7th 9:30 – 11:20 LINC 368
closed book.

So far

Our two sample comparisons have focused on means (or proportions)

What else could we compare?

- Medians
- Variances
- Whole distributions

Comparing medians: Mood's median test

Mood's median test

Setting: two indpendent samples

Y_i i.i.d sample of size n from popuation with c.d.f F_Y
X_i i.i.d sample of size m from popuation with c.d.f F_X

$m_Y = F_Y^{-1}(0.5) =$ median of population that Y is sampled from.
$m_X = F_X^{-1}(0.5) =$ median of population that X is sampled from.

Comparison of interest: Is m_Y the same as $m_{\mathbf{X}}$?

Example

A study is performed to assess the effect of fish oil supplements on diastolic blood pressure

imagine 12 from Fish Oil population

- 25 subjects are randomly assigned to receive fish oil ($n_Y = 12$) or regular vegetable oil ($n_X = 13$) for two weeks.
- Each subject's decrease in diastolic blood pressure over those two weeks is recorded (bigger numbers => better reduction in blood pressure)

good reduction

Fish oil: *-2.2, -0.8, 3.7, 4.9, 5, 5.2, 5.3, 6, 8, 8, 10.4* and *14*

Regular oil: *-6.4, -6.4, -5.9, -5.8, -5.3, -4.9, -4.4, 0.2, 2.1, 2.5, 2.5, 6.1* and *8.9*

Question: Is the median blood pressure reduction the same for these two treatments?

→ mean?

Your turn

$H_0 : m_Y = m_X = m$

- If the null is true, $m_Y = m_X = m$, what is our best guess for the median m? 1) Combine data, get median 2) average sample medians
- If the null is true, what proportion of the sample from Y should be larger than m? $\sim \frac{1}{2}$
- If the null is true, what proportion of the sample from X should be larger than m? $\sim \frac{1}{2}$

Estimating the combined median

$$\hat{m}_Y = \hat{m}_X \underset{\text{common}}{=} \hat{m} = \text{median}(Y_1, Y_2, \ldots, Y_n, X_1, X_2, \ldots, X_m)$$

If the null is true, this estimate is an unbiased and consistent estimate of the common median, m.

We expect $P(Y_i > m) = P(X_i > m). = 0 \cdot 5$

For large samples, $P(Y_i > \underset{\substack{\uparrow \\ \text{estimate}}}{\hat{m}}) = P(X_i > \hat{m}) = 0.5$

Mood's median test

Procedure:

1. Find the combined median $\hat{m}$.
2. Test the true proportion of Y's greater than $\hat{m}$ is equal to the true proprtion of X's greater than $\hat{m}$.
    - Z-test for proportions/Chi-square test or Fishers exact test

Example cont.

Combined sample:

```
##  [1] -6.4 -6.4 -5.9 -5.8 -5.3 -4.9 -4.4 -2.2 -0.8
## [10]  0.2  2.1  2.5  2.5  3.7  4.9  5.0  5.2  5.3
## [19]  6.0  6.1  8.0  8.0  8.9 10.4 14.0
```

R output

25 numbers

13th largest

sample median

Combined median, $\hat{m} = 2.5$

	Number $> \underset{2.5}{\hat{m}}$	Number $\leq \hat{m}$
Fish Oil	10	2
Regular Oil	2	11

Example cont.

$$Z = \frac{\hat{p}_Y - \hat{p}_X}{\sqrt{\hat{p}_c(1-\hat{p}_c)\left(\frac{1}{n}+\frac{1}{m}\right)}}$$

$$= \frac{\frac{10}{12} - \frac{2}{13}}{\sqrt{\frac{12}{25}(1-\frac{12}{25})\left(\frac{1}{12}+\frac{1}{13}\right)}}$$

$$= 3.4$$ compare $N(0,1)$

p-value $= 6.8 \times 10^{-4}$. 0.00068

There is convincing evidence that the median BP reduction on fish oil is different to the median BP reduction on regular oil.

Wilcoxon Rank Sum test

Wilcoxon Rank Sum

Wilcoxon Rank Sum, a.k.a Mann-Whitney U-test

Often presented as a test for equality of medians, like Wilcoxon Signed Rank, **this isn't true without further assumptions**.

Wilcoxon Rank Sum Procedure

1. Combine the samples
2. Rank the observations in the combined sample from smallest (1) to largest ($n + m$). If there are ties, assign the average rank to the tied observations.
3. **Test statistic:** Sum of the ranks in the sample with the smaller sample size
   ↑ not crucial
4. **p-value:** either use Normal approximation, or via permutation
   large samples
   ↓ permute group labels on the ranks,

Intutition: if all the observations come from the same distribution, it would be unlikely for all the observations in the samller sample to have all the highest ranks (or lowest).

Example

Combined sample:

```
## Regular Oil Regular Oil Regular Oil Regular Oil
##        -6.4 ~~1~~ 1.5  -6.4 ~~2~~ 1.5  -5.9 3   -5.8 4
## Regular Oil Regular Oil Regular Oil    Fish Oil
##        -5.3 5      -4.9 6      -4.4 7      -2.2 8
##    Fish Oil Regular Oil Regular Oil Regular Oil
##        -0.8 9       0.2 10      2.1 11      2.5 ~~12~~ 12.5
## Regular Oil    Fish Oil    Fish Oil    Fish Oil
##         2.5 ~~13~~ 12.5  3.7 14  4.9 15      5.0 16
##    Fish Oil    Fish Oil    Fish Oil Regular Oil
##         5.2 17      5.3 18      6.0 19      6.1 20
##    Fish Oil    Fish Oil Regular Oil    Fish Oil
##         8.0 ~~21~~ 21.5  8.0 ~~22~~ 21.5  8.9 23  10.4 24
##    Fish Oil
##        14.0 25
```

```
## [1] 208
```

$\boxed{208} = 8 + 9 + 14 + \ldots + 24 + 25$

Test statistic → HW#8 you will compare to reference

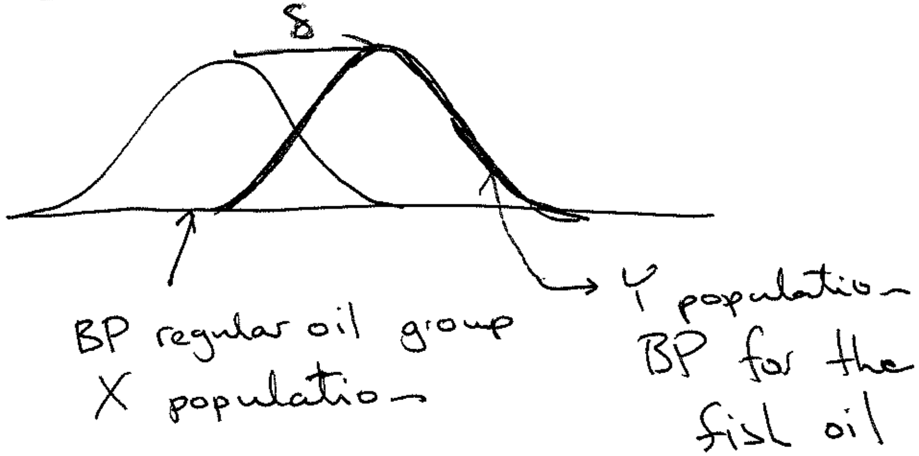
- Ranks from 1 to 25
- Resolve ties by averaging ranks

Fish oil $n = 12$

Regular $m = 13$

Problems

- Location-shift assumption



Y : Shift X distribution δ units

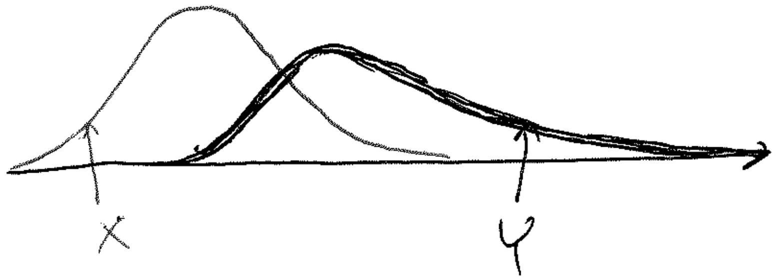
“additive effect”

$\mu_Y = \mu_X + \delta$

$m_Y = m_X + \delta$

$10^{th}\text{ percentile}_Y = 10^{th}\text{ percentile}_X + \delta$

- Not location shift



Mean is different

Medians are different

Variances and different

Shape is different

Wilcoxon Rank Sum

- Assume location-shift
  "additive treatment"

  $H_0: \delta = 0 \quad \mu_X = \mu_Y,\ m_X = m_Y$

  $H_A: \delta \neq 0$

  Wilcoxon Rank Sum is exact
  consistent

- If ~~you don't~~
  you don't assume location shift

  Wilcoxon Rank Sum is not:
  - a test for same means
  - a test for same medians

  - WR Sum is ~~ont~~ not exact, or consistent

~~Exact~~ Consistent for the hypothesis

$H_0: P(Y > X) = 0.5$