Properties of difference in sample means

ST551 Lecture 18

Charlotte Wickham

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Let $Y_1, \ldots, Y_n$ be an i.i.d sample of size n from a population with mean μ_Y and variance σ_Y^2, and
let $X_1, \ldots, X_m$ be an i.i.d sample of size m from a population with mean μ_X and variance σ_X^2.
The samples are drawn indpendently of each other.

Q1 Using the Central Limit Theorem, find the approximate distribution of $\overline{Y}$ and $\overline{X}$ for large sample sizes.

$\overline{Y} \dot\sim$

$\overline{X} \dot\sim$

Q2 $\overline{Y}$ and $\overline{X}$ are independent. Justify this fact.

Q3 Derive the distribution of $\overline{Y} - \overline{X}$. A useful fact (from earlier in the quarter) is provided in the box below.

If $X \sim N(\mu_X, \sigma_X^2)$ and $Y \sim N(\mu_Y, \sigma_Y^2)$, independent of X.

Then,

$$Z = X + Y \sim N(\mu_X + \mu_Y, \sigma_X^2 + \sigma_Y^2)$$

$\overline{Y} - \overline{X} \dot\sim$

Q4 Using your result from Q3, suggest a test statistic for testing the null hypothesis $H_0 : \mu_Y - \mu_X = \delta_0$ that would have a (approximately) **standard** Normal distribution, when the null hypothesis is true.